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TECHNICAL NOTES

A modified formula for calculating the heat transfer coefficient by the shadowgraph technique

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INTRODUCTION

IT IS WELL known that optical techniques are widely used in the measurement of fluid flow and heat transfer. These techniques include the Schlieren, shadowgraph, interferometric methods, etc., all of which are described in detail in the open literature [1–3]. Optical measurements have many advantages over other techniques. Perhaps the most important one is the absence of an instrument probe that could influence the flow field and temperature field, thus allowing the study of rapid transients since the light beam is considered as essentially inertialess. The sensitivity of optical methods is quite different and they can be used to study a variety of systems. Interferometers are often used to study flows and heat transfer in which the density gradient (temperature gradient) is small, while the Schlieren and shadowgraph systems are often employed to study the flow and heat transfer in which larger density gradients (temperature gradients) are present. Initially, the shadowgraph technique was rarely used for quantitative density (temperature) measurements. Correspondingly, it was considered one of the tools for 'flow visualization'. However, continuous efforts have been made to develop a quantitative shadowgraph method [4–8]. On the other hand, since the interferometer cannot be applied in cases such as melting [9–11], the shadowgraph method must be used. This is due mainly to a large index of refraction variation of the test media.

Generally, the shadowgraph measurement is basically integral, meaning that it integrates the quantity measured over the length of the light beam. Therefore, the shadowgraph method is well suited for measurements in two-dimensional fields where there is no index of refraction or density variation in the field along the light beam except at the beam's entrance to and exit from the test region. When the shadowgraph method is used quantitatively, the index of refraction variation at the entrance and exit of the test region must be considered and an appropriate end correction must be made. However, to the authors' knowledge, no such correction appears in the open literature. Thus, the purpose of this work is to develop a modified formula for calculating the heat transfer coefficient in which the index of refraction variation at the exit of the test region is taken into account based on a practical shadowgraph system. In addition, an example is given to examine the error between the results obtained by using the modified formula and the non-modified formula. It is shown that the modified formula should be employed to calculate the heat transfer coefficient when the shadowgraph is used quantitatively.

THE SHADOWGRAPH TECHNIQUE

The principle of the shadowgraph is described in detail by Hauf and Grigull [3]. It is cited here only for convenience. A diagram illustrating the shadow system is shown in Fig. 1. According to this figure, a heated surface has a length, L, and a screen located a distance, L_z, far from the center of the heated surface. When a light beam passes through the region above the surface (test region), refraction will occur due to the different temperature gradients in this region. The light beam near the wall will have a maximum refraction and, eventually, the corresponding displacement on the screen has a maximum value which in turn is used to determine both the temperature gradient on the wall and the heat transfer coefficient. A coordinate system, as shown in Fig. 1, has been selected. It is assumed that the index of refraction of the media in the test cell varies more rapidly in the y-direction than in the z-direction. Therefore, the index of refraction variation in the z-direction is neglected, which implies $n = n(y)$. Generally, the slope of the trajectory of the light beam is very small. It is further assumed that $\mathrm{d}n/\mathrm{d}y =$ constant. Based on the foregoing discussion, the trajectory equation can be written as

$$z = \int_{y_{01}}^{y} \frac{\mathrm{d}y}{2n'/n_0(y-y_{01})^{1/2}}, \tag{1}$$

where n' is the first derivative of the index of refraction with respect to y. Integration of equation (1) yields

$$y_0 - y_{01} = \frac{n'L^2}{2n_0}. \tag{2}$$

The slope of the trajectory at the outlet of the test region is calculated as

$$y_0' = \frac{n'L}{n_0} = \frac{y_0 - y_{01}}{L/2}. \tag{3}$$

The displacement on the screen is as follows:

$$y = \frac{1}{n_0}\frac{\mathrm{d}n}{\mathrm{d}y}LL_z + y_{01}. \tag{4}$$

When the light beam passes near the wall, $y_{01} = 0$ and $n_0 = n_w$. Thus, the maximum displacement on the screen, y_{max}, can be obtained as

$$y_{max} = \frac{1}{n_w}\frac{\mathrm{d}n}{\mathrm{d}y}LL_z. \tag{5}$$

Now, since the index of refraction is a function of the temperature, one can write

$$\frac{\mathrm{d}n}{\mathrm{d}y} = \frac{\mathrm{d}n}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}y}. \tag{6}$$

Substitution of equation (6) into equation (5) gives the temperature gradient on the wall as

$$\left.\frac{\mathrm{d}t}{\mathrm{d}y}\right|_w = \frac{y_{max} n_w \left.\frac{\mathrm{d}t}{\mathrm{d}n}\right|_w}{LL_z}. \tag{7}$$

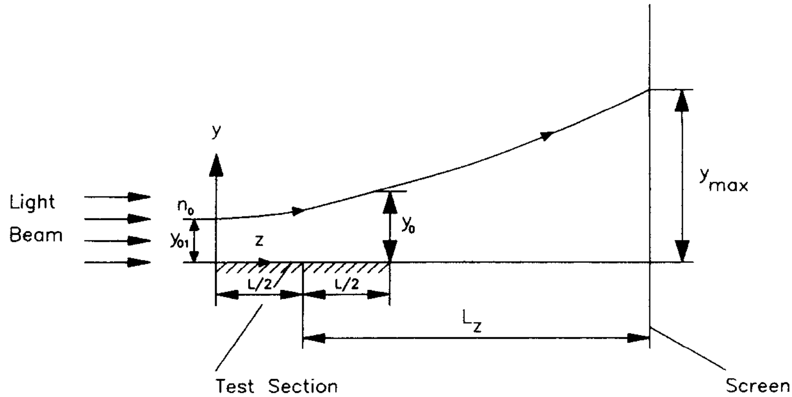

FIG. 1. Schematic of the basic shadowgraph system.

In the above equation, subscript w refers to the heated surface wall values. The heat transfer coefficient, h, and Nusselt number, Nu, can be defined as

$$h = -\frac{k}{t_w - t_f}\frac{dt}{dy}\bigg|_w$$

and

$$Nu = \frac{hR}{k}, \tag{8}$$

where t_w, t_f, R, and k are the wall temperature, fluid temperature, characteristic length, and thermal conductivity of the fluid, respectively.

Therefore, using equation (6), equation (8) can be rewritten as

$$Nu = -\frac{R}{t_w - t_f}\frac{n_w y_{max}}{LL_z}\frac{dt}{dn}\bigg|_w \tag{9}$$

This is the formula for calculating the Nusselt number on the wall in the basic shadowgraph system.

MODIFIED CASE

The practical shadowgraph system is not the same as the one shown in Fig. 1. Usually, several layers of different optical materials are used to cover the test region, which reduces the heat exchange between the test section and the ambient. Thus, after the light beam passes through the test region, it will continue to pass through to the outside of the optical medium until it reaches the screen. However, this medium has a different index of refraction from that of the test region, which eventually influences the displacements on the screen. If equation (9) is still used to calculate the Nusselt number, a misleading result will be obtained. Here, we consider a practical system, as illustrated in Fig. 2. There are five layers of different optical media outside of the test section. The indexes of refraction, $n_1, n_2, \ldots, n_5$ in the medium outside of the test region, with thicknesses of $d_1, d_2, \ldots, d_5$, respectively, are assumed to be constant. The length, L, of the heated surface is the same as shown in Fig. 1. It is also assumed that the light beam enters the test region parallel to the inlet. From the law of refraction

$$n_0 \sin\alpha_0 = n_1 \sin\alpha_1 = \cdots = n_5 \sin\alpha_5. \tag{10}$$

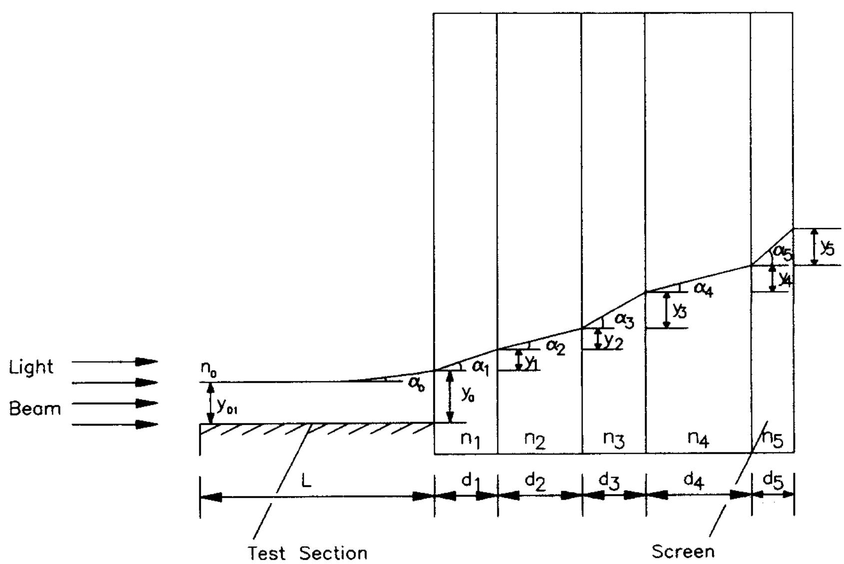

FIG. 2. Schematic of the practical shadowgraph system.

Using the triangle function correlation, one can write

$$\sin \alpha_i = \frac{\tan \alpha_i}{\sqrt{(1+\tan^2 \alpha_i)}} \tag{11}$$

where α_i is the refraction angle of the light beam in the ith medium, as shown in Fig. 2. Also from Fig. 2

$$\tan \alpha_0 = \frac{y_0 - y_{01}}{L/2} \tag{12}$$

$$\tan \alpha_i = \frac{y_i}{d_i}. \tag{13}$$

Therefore, the maximum displacement on the screen is

$$y_{\max} = y_0 + \sum_{i=1}^{5} y_i. \tag{14}$$

From equations (10) and (11)

$$\frac{\tan \alpha_i}{\sqrt{(1+\tan^2 \alpha_i)}} = \frac{n_0}{n_i} \frac{\tan \alpha_0}{\sqrt{(1+\tan^2 \alpha_0)}}. \tag{15}$$

Simplification of equation (15) yields

$$\tan \alpha_i = \frac{\dfrac{n_0}{n_i} \dfrac{\tan \alpha_0}{\sqrt{(1+\tan^2 \alpha_0)}}}{\sqrt{\left(1 - \left(\dfrac{n_0}{n_i}\right)^2 \dfrac{\tan^2 \alpha_0}{1+\tan^2 \alpha_0}\right)}}. \tag{16}$$

Substitution of equation (16) into equation (13) results in

$$y_i = \frac{d_i \dfrac{n_0}{n_i} \dfrac{\tan \alpha_0}{\sqrt{(1+\tan^2 \alpha_0)}}}{\sqrt{\left(1 - \left(\dfrac{n_0}{n_i}\right)^2 \dfrac{\tan^2 \alpha_0}{1+\tan^2 \alpha_0}\right)}}. \tag{17}$$

Since

$$\tan \alpha_0 = y_0' = \frac{n'}{n_0} L, \tag{18}$$

when the light beam near the wall is considered, then

$$y = y_{\max}, \quad n_0 = n_w, \quad y_{01} = 0. \tag{19}$$

Substitution of equations (5), (17), (18), and (19) into equation (14) yields

$$y_{\max} = \frac{n'}{n_w} \frac{L^2}{2} + \frac{n_w \dfrac{n'L}{n_w}}{\sqrt{\left(1 + \left(\dfrac{n'L}{n_w}\right)^2\right)}} \sum_{i=1}^{5} \frac{\dfrac{d_i}{n_i}}{\sqrt{\left[1 - \dfrac{\dfrac{n_w}{n_i}\left(\dfrac{n'L}{n_w}\right)^2}{1+\left(\dfrac{n'L}{n_w}\right)^2}\right]}}. \tag{20}$$

The above equation is very complicated for computing $y_{\max}$. However, according to Hauf and Grigull [3], the scale order of $(n'L/n_w)$ is about 1/100. Therefore, the term $(n'L/n_w)^2$ in equation (20) can be neglected. After rearranging, this will lead to

$$y_{\max} = \frac{n'L}{n_w} \frac{L}{2} + \frac{n'L}{n_w} \sum_{i=1}^{5} \frac{n_w d_i}{n_i}. \tag{21}$$

Now, considering $n' = dn/dt \; dt/dy$, then

$$\left.\frac{dt}{dy}\right|_w = \frac{\left.\dfrac{dt}{dn}\right|_w \dfrac{n_w}{L} y_{\max}}{\dfrac{L}{2} + \sum_{i=1}^{5} \dfrac{n_w d_i}{n_i}}, \tag{22}$$

and the Nusselt number in the practical system, Nu_p, is

$$Nu_p = -\frac{\dfrac{R}{t_w - t_f} \dfrac{n_w y_{\max}}{L} \left.\dfrac{dt}{dn}\right|_w}{\dfrac{L}{2} + \sum_{i=1}^{5} \dfrac{n_w d_i}{n_i}}. \tag{23}$$

Comparison of equation (23) with equation (9) indicates that the relative error, Δ, between these two equations is defined by

$$\Delta = 1 - \frac{Nu}{Nu_p} = \frac{\sum_{i=1}^{5} \left(1 - \dfrac{n_w}{n_i}\right) \dfrac{d_i}{L/2}}{1 + \sum_{i=1}^{5} \dfrac{2d_i}{L}}. \tag{24}$$

Inspection of equation (24) indicates that the error is dependent on n_w/n_i and d_i/L. When d_i/L is determined, the error is a function of n_w/n_i. However, it often varies with the temperature and it can be concluded that the temperature is the main factor affecting the sensitivity of the shadowgraph technique.

EXAMPLE

The shadowgraph technique is used to calculate the heat transfer coefficient on the vertical wall when melting in a rectangular cavity is in process [12]. The dimensions of the cavity are 100 mm high, 60 mm wide, and 50 mm deep. The two layers of optical glass 6 mm thick were covered on the front and back sides respectively. In addition, there is 10 mm air gap between these two layers of optical glass in order to reduce the heat exchange between test section and ambient. The screen, of 5 mm thickness, on which the shadowgraph is formed, was made of Plexiglas and located 25 mm from the optical glass. In the experiment n-eicosane is used as phase change material because it is transparent in the liquid phase. Due to the large index of refraction variation with temperature, the shadowgraph method had to be used quantitatively. The schematic of the shadowgraph system is given in Fig. 2. The thickness and index of refraction of five layers of optical medium outside of the test section (rectangular cavity) are: $d_1 = d_3 = 6$ mm, $d_2 = 10$ mm, $d_4 = 25$ mm, $d_5 = 5$ mm, $n_1 = n_3 = 1.516$, $n_2 = n_4 = 1$, and $n_5 = 1.49$. The relation of refraction index of n-eicosane with temperature is given by

$$n = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \tag{25}$$

where

$$a_0 = 1.45300, \qquad a_1 = -6.33377 \times 10^{-4}$$
$$a_2 = 7.00151 \times 10^{-6}, \quad a_3 = -6.66836 \times 10^{-8}.$$

The temperature of the heated surface can be adjusted to the requirement for different melting cases. The shadowgraph on the screen was taken by a camera and the maximum displacement of the light beam near the vertical wall was measured and used to calculate the Nusselt number. Equations (9) and (23) are employed to find the difference. It can be seen that the deviation in the results given in equations (9) and (23) varies with temperature. The relative error is shown in Fig. 3, which is drawn based on the data generated by equation (24). In this case, when the temperature is varied within 40–100°C, the error will be between 16 and 18%. Therefore, it is concluded that the modified formula should be applied in the case of quantitative use of the shadowgraph method.

CONCLUSION

In this paper, practical use of the shadowgraph technique is considered and a more accurate formula, equation (23), is

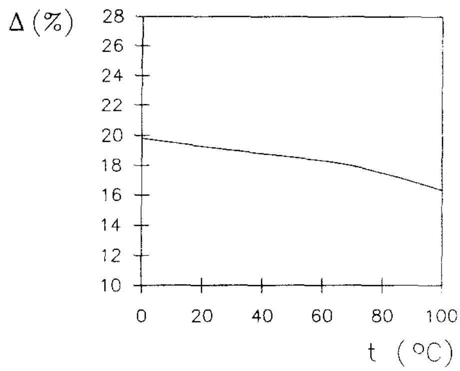

FIG. 3. Error vs temperature.

developed for calculating the Nusselt number on the wall. This formula includes both the effects of the test section configuration and the effects of the ratio of the index of refraction of the test material to the index of refraction of the materials outside the test cell. The relative error can be estimated by equation (24). Generally, the index of refraction of the optical medium in the test section often varies with the temperature. Thus, the error induced also depends on the temperature, when the test section configuration and measuring system are determined. Therefore, the use of equation (23) is suggested, rather than equation (9), when the shadowgraph method is used quantitatively. Although five layers of the medium outside of the test cell are taken into account in this paper, equation (23) can be applied to other cases with any number of different optical materials outside the test section; for example, m layers. In such cases, the upper limit of summation in equations (23) and (24) need to be changed to m instead of 5.

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REFERENCES

1. R. J. Goldstein, Optical techniques for temperature measurements. In *Measurements in Heat Transfer*. Hemisphere, Washington, DC (1976).
2. R. J. Goldstein, Optical systems for flow measurement: shadowgraph, Schlieren, and interferometric techniques. In *Fluid Mechanics Measurements*. Hemisphere, Washington, DC (1983).
3. W. Hauf and U. Grigull, Optical method in heat transfer. In *Advances in Heat Transfer*, Vol. 6. Academic Press, New York (1970).
4. M. S. Uberoi and L. S. G. Kovasznay, Analysis of turbulent density fluctuations by the shadow method, *J. Appl. Phys.* **26**, 19–24 (1955).
5. M. R. Davis, Measurements in a subsonic jet using a quantitative Schlieren technique, *J. Fluid Mech.* **46**, 631–656 (1971).
6. L. Hesselink and B. S. White, Digital image processing of flow visualized photographs, *Appl. Opt.* **22**, 1454–1461 (1983).
7. T. Grandke, Theory and application of the laser shadow technique, part 1. Two dimensional turbulent flow, *Exp. Fluids* **3**, 77–85 (1985).
8. T. Grandke, Theory and application of the laser shadow technique, part 2. Axisymmetric turbulent flow, *Exp. Fluids* **3**, 277–282 (1985).
9. R. D. White, A. G. Bathelt, W. Leindenfrost and R. Viskanta, Study of heat transfer and melting from a cylinder embedded in a phase change material, ASME Paper 77-HT-42 (1977).
10. A. G. Bathelt and R. Viskanta, Heat transfer at the solid–liquid interface during melting from a horizontal cylinder, *Int. J. Heat Mass Transfer* **23**, 1493 (1980).
11. Q. J. Wang and W. Y. Li, Heat transfer and interface motion in the presence of subcooling for melting around a horizontal tube with and without axial fins, advance in phase change heat transfer, *Proc. Int. Symp. on Phase Change Heat Transfer* (Edited by M. D. Xin), p. 350 (1988).
12. Z. F. Dong, Research on solid–liquid phase change heat transfer in a rectangular cavity with phase change material—contact melting study, Ph.D. Thesis, Xi'an Jiaotong University, China (1988).

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An improved velocity field for the Madejski splat-quench solidification model

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RAPID solidification processing (RSP) consists of the production of materials from the melt, at high rates of cooling and freezing, in order to achieve certain desired or perhaps unusual characteristics. Many forms of RSP are currently in use, one of the most elementary being the 'splat-quenching' of liquid-metal drops by impact onto a solid substrate. A mathematical model of the splat-quench process was developed some time ago by Madejski [1, 2] and he obtained reasonably good agreement with experiment. In fact, his model is still finding use [3] as an aid in the interpretation of experimental splat-quenching data. In this note, we make a substantial improvement upon the velocity field used by Madejski and demonstrate that the model obtained therewith is correspondingly improved.

Madejski postulated a velocity field for the unsolidified portion of the liquid drop as it spreads on the substrate. He then required that the time derivative of the mechanical plus interfacial energy of the drop be zero. The velocity field